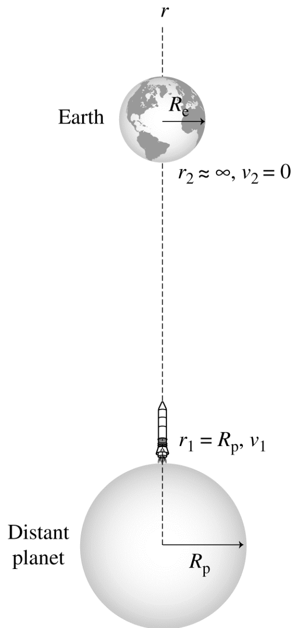
12.19. Model: Model the distant planet (p) and the earth (e) as spherical masses. Because both are isolated, the mechanical energy of the object on both the planet and the earth is conserved.

Visualize:



Let us denote the mass of the planet by M_p and that of the earth by M_e. Your mass is m_0. Also, acceleration due to gravity on the surface of the planet is g_p and on the surface of the earth is g_e. R_p and R_e are the radii of the planet and the earth, respectively.

Solve: **(a)** We are given that $M_P = 2M_e$ and $g_p = \frac{1}{4}g_e$.

Since $g_p = \dfrac{GM_p}{R_p^2}$ and $g_e = \dfrac{GM_e}{R_e^2}$, we have

$$\frac{GM_p}{R_p^2} = \frac{1}{4}\frac{GM_e}{R_e^2} = \frac{1}{4}\frac{G(M_p/2)}{R_e^2} \Rightarrow R_p = \sqrt{8}R_e = \sqrt{8}(6.37\times 10^6\ \text{m}) = 1.80\times 10^7\ \text{m}$$

(b) The conservation of energy equation $K_2 + U_2 = K_1 + U_1$ is

$$\frac{1}{2}m_o v_2^2 - \frac{GM_p m_o}{r_2} = \frac{1}{2}m_o v_1^2 - \frac{GM_p m_o}{R_p}$$

Using $v_2 = 0$ m/s as $r_2 \to \infty$, we have

$$\frac{1}{2}m_o v_{\text{escape}}^2 = \frac{GM_p m_o}{R_p}$$

$$\Rightarrow v_{\text{escape}} = \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{2G(2M_e)}{R_p}} = \sqrt{\frac{4(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\ \text{kg})}{1.80\times 10^7\ \text{m}}} = 9410\ \text{m/s}$$